7. (10%) Suppose we have the following stage-type device (an example of the structure introduced by Cox) as a service facility. Notice that this has an infinite number of exponential stages, all at the rate μ. Find an explicit expression for b(x).

second system is an M/M/2 system with input rate λ and each server with rate μ.
(3%)(a) Find the mean response time, T, for the first system.
(10%)(b) For the second system (M/M/2), find
(2%)(i) The utilization, ρ.
(2%)(ii) The equilibrium probabilities P_k in terms of P_0.
(2%)(iii) P_0.
(2%)(iv) The average number in system, $\overline{N}$.
(2%)(v) The mean response time, T.
(2%)(c) Compare the value of T found for the two systems and prove which system has a smaller mean response time.

5. (16%) Consider an M/M/1 system in which no queue is allowed (i.e. M/M/1/1 pure loss). A customer who arrives to find the server busy will be "lost" (i.e. leave immediately). However, for the purpose of this problem, any customer who leaves (lost or served) will be considered a "departure from the system"; customers who leave after being served will be considered a "departure from the server".
(5%)(a) Let $\tilde{d}_1$ = time between "departures from the server". Find $D_1^*(s)$, the Laplace transform of the p.d.f. of $\tilde{d}_1$.
(4%)(b) Let $\tilde{d}_2$ = time between "departures from the system". Find P_0 = fraction of "departures from the system" which are "departures from the server".
(7%)(c) Find the Laplace transform, $D_2^*(s)$, of the p.d.f. of $\tilde{d}_2$.

6. (19%) Consider an M/M/1 queueing system with bulk arrivals and bulk service as follows. Arrival instants occur at a rate λ, BUT, at each such instant *two* customers arrive. Service occurs at a rate μ, BUT, at each service completion, the man who was in service leaves *and* with probability 2/3, the man at the head of the queue (if any) will also leave at the same time (he gets a "free ride").
(5%)(a) Draw a completely labelled state diagram for the number of customers in system.
(5%)(b) Write down the full set of equations describing the relationship among the equilibrium state probabilities, P_k.
(6%)(c) Find an explicit expression for $P(Z) \equiv \sum_{k=0}^{\infty} P_k Z^k$ in terms of P_0, P_1, λ, μ.
(3%)(d) Find an explicit relation between P_0 and P_1.

Sample Midterm for Queueing Theory (closed book)

Instructor: Professor Ying-Dar Lin

1. (12%) Consider the two sequence $f(n)$ and $g(n)$ given as follows:

$$f(n) = A\alpha^n, n = 0, 1, 2, ...$$
$$g(n) = 1, n = 0, 1, 2, ...$$

where $\alpha < 1$ and $f(n) = g(n) = 0$ for $n < 0$.
(6%)(a) Find the convolution of $f(n)$ with $g(n)$ without using transforms.
(6%)(b) Repeat part (a) using transforms.

2. (13%) Consider the two-state Markov chain below:
(10%)(a) Find the state dependent probabilities $\Pi(n) = [\pi_1(n), \pi_2(n)]$.
(3%)(b) Find the equilibrium probabilities $\Pi = [\pi_1, \pi_2]$.

3. (15%) Consider a birth-death process for which

$$\lambda_k = (k+2)\lambda, k = 0, 1, 2, ...$$
$$\mu_k = k\mu, k = 0, 1, 2, ...$$

(3%)(a) Draw the fully labeled state diagram.
(4%)(b) Write down the set of equation which must be satisfied by $P_k(t) = $ P[k in system at time t].
(4%)(c) Find the equilibrium probability P_k in terms of P_0.
(4%)(d) Find P_0.

4. (15%) We wish to compare the mean response time for the two queueing system shown below. They both have the same total arrival rate of λ customers per second (from a Poisson arrival process) and they each have two exponential servers, each at a rate μ. The first system is really two independent M/M/1 systems each with input rate $\lambda/2$ and service rate μ. The