NOT WRITE DOWN (OR SOLVE) THE STATE EQUATIONS.

5. (8%) Consider an M/M/1 computer system with arrival rate λ (jobs/sec). Each job has an average of $1/\mu$ operations (distributed exponentially). The computer has a capacity of C operations per second.
(2%)(a) What is the expression for ρ?
(2%)(b) What is the expression for the mean response time T?
(2%)(c) For the rest of the problem, we assume $\mu = 1$. If we vary λ and C in a way that holds T fixed (at T_0)then how does C vary with λ. Give the expression.
(2%)(d) For ρ=1/2 and λ=0.1, find T.

6. (22%) Consider the $M/E_2/1$ queueing system and solve it using the method of stages due to Erlang. Let λ be the arrival rate, and $1/\mu$ be the mean service time.
(2%)(a) Find the second moment of service time.
(2%)(b) Find the utilization factor ρ.
(2%)(c) What is the (two-dimentional) state description of this system?
(2%)(d) Draw a fully labeled state disgram for the Markov chain described in part (c).
(2%)(e) What is the (one-dimensional) state description of this system? (2%)(f) Draw a fully labeled state diagram for the Markov chain described in part (e).
(4%)(g) Let P_j=P[system in state j] for the state as defined in part (e). Write down the full set of balance equations for this chain.
(6%)(h) Let $P(Z) = \sum_{j=0}^{\infty} P_j Z^j$. Find an explicit expression for $P(Z)$.

7. (11%) Let $B^*(s)$ be the Laplace transform for the service time pdf as given below:

$$B^*(s) = \frac{a(s+2)}{(s+1/2)(s+4)}.$$

(2%)(a) Find the value for a.
(2%)(b) Find the pdf $b(x)$ corresponding to this transform.
(3%)(c) Draw a fully labelled diagram of a series/parallel service box which generates this service time pdf.
(4%)(d) Repeat part (c) for a Coxian service box.

Midterm for Queueing Theory (closed book) 4/28/1994

Instructor: Professor Ying-Dar Lin

1. (10%) Consider a function $f(t)$ for which $f(t) = 0$ for $t < 0$. Its Laplace transform is given by

$$F^*(s) = \frac{s^2 + 11s + 13}{s^2 + 5s + 4}.$$

Find $f(t)$.

2. (18%)Consider an M/M/1 queueing system with service rate μ. Let N = number in system. For this system we have an arrival rate λ if $N < K$. If $N \geq K$, then the arrival rate is $\alpha\lambda$ where $0 \leq \alpha$.
(4%)(a) Draw a fully labelled state diagram.
(10%)(b) Find P_k explicitly in terms of λ, μ, K and α.
(2%)(c) Find ρ for this system.
(2%)(d) What condition among the parameters will guarantee stability?

3. (12%) Now consider a regular M/M/1 system with service rate μ and arrival rate λ, with $\lambda < \mu$. Recall our notation where $\tilde{t}$ is the interarrival time with p.d.f. $a(t)$ and Laplace transform $A^*(s)$. For the service time, $\tilde{x}$, we have $b(x)$ and $B^*(s)$. Let $\tilde{d}$ be the interdeparture time with p.d.f. $d(t)$ and transform $D^*(s)$. For M/M/1, it is known that $1 - \rho$ = P[departure leaves behind an empty system].
(2%)(a) Express $\tilde{d}$ in terms of $\tilde{t}$, $\tilde{x}$, and ρ.
(2%)(b) Express $d(t)$ in terms of $a(t)$, $b(t)$, and ρ. (hint: use conditional prob and convolution)
(2%)(c) Express $D^*(s)$ in terms of $A^*(s)$, $B^*(s)$, and ρ.
(2%)(d) For this M/M/1 system, what are the expressions for $A^*(s)$ and $B^*(s)$?
(2%)(e) From (c) and (d), find $D^*(s)$ explicitly.
(2%)(f) From (e), find $d(t)$ explicitly.

4. (19%) We now consider the Jackson type open queueing network shown below:
All stations have single servers with exponential distributions, where station i has parameter μ_i. And all external arrival proccesses are Poisson.
(5%)(a) Find λ_1 and λ_2.
(4%)(b) Find $P(n_1, n_2)$=$P[n_1$ customers in station one and n_2 customers in station two].
(4%)(c) Find R, the rate at which customers depart from the network after being served at station one.
(6%)(d) Let (i, j) be the state description, where i is the number of customers in node 1 and j is the number of customers in node 2. Draw a fully labelled state diagram for the network. DO